Identification, Uncertainty Characterization and Robust Control Synthesis Applied to Large Flexible Structures Control

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Abstract

This paper demonstrates an approach to frequency domain identification for the explicit purpose of designing robust H_∞ controllers. The approach transforms raw experimental data into a plant set estimate directly usable by modern robust control design software (e.g., Matlab Robust Control Toolboxes [1][2]). A key issue in control design from raw data is the question of whether the controller will work when applied to the true system. The main feature of this approach is that the resulting controller is guaranteed to work as designed (when applied to the true system) to a prescribed statistical confidence. While the overall methodology addresses key theoretical issues, it has at the same time been specifically designed to support practical implementations. A simulation example is included the demonstrate the overall approach.

1 Introduction

The goal of robust control design is to synthesize a controller which establishes certain closed-loop properties (e.g., stability, performance, sensitivity reduction, etc.) for a specified set of open-loop plants. The set of open loop plants is typically characterized using a priori information concerning the physics of the system, system modelling, engineering judgement, experience with similar systems, etc.

In the interest of reducing conservatism in the plant uncertainty description, there have been recent efforts aimed at characterizing the plant set using system identification techniques [5][6][8][13][16][17][19][22][24][25][30]. In the case that experimental input/output data is available from the system, this requires characterizing the set of plants which are consistent with (or equivalently, can't be discounted based on), the data.

For the purposes of this paper, a plant $P(z^{-1})$ will be identified in the representation shown in Figure 1. Robust control methods are then applied to uncertainty expressed in this form. Here, $\hat{P}(z^{-1})$ is a nominal estimate of the true plant $P(z^{-1})$; Δ_A is the additive uncertainty defined as $\Delta_A = P - \hat{P}$; $C(z^{-1})$ is the digital controller under consideration, $d(k)$ is a white Gaussian noise disturbance of unit variance, and $W_d(z^{-1})$ is a frequency weighting filter which characterizes the effect of $d(k)$ on the open-loop plant output $y(k)$.

For control design purposes, the additive uncertainty is identified in the form $\Delta_A = \Delta W_A$ such that Δ is norm bounded, i.e., such that $\|\Delta\|_\infty \leq 1$. The filter W_A is then typically incorporated into the control design, to ensure robustness properties over the additive uncertainty set. If desired, the disturbance spectrum W_d can also be estimated. The overall processing scheme is summarized in Figure 2.

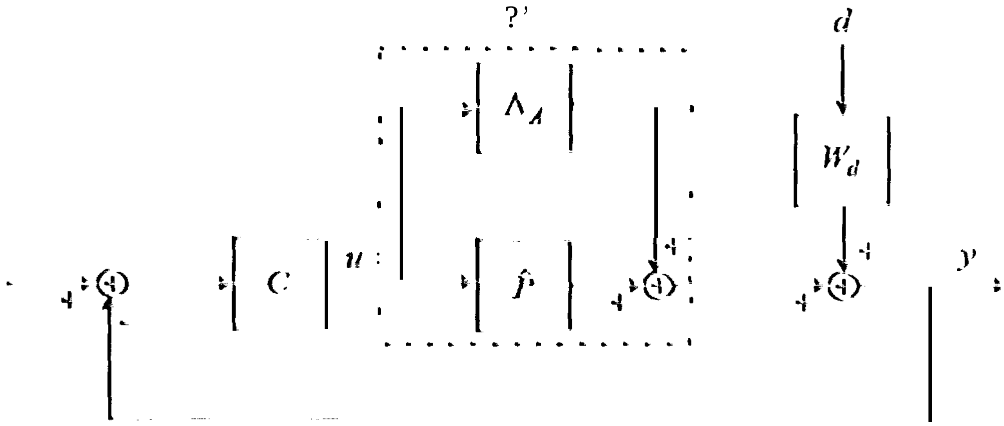

Figure 1: Canonical representation for identification and robust control

A more complete treatment of the approach used in this paper can be found in Bayard and Yam [6]. Also, each of the constituent algorithms have since appeared in the literature, cf., [3][5][4][7][28].

2 Frequency Domain Estimation

Consider the single-input single-output system of the form,

$$y(k) = P(z^{-1})u(k) + W_d(z^{-1})d(k) \tag{1}$$

where W_d is a minimal phase rational noise transfer function in the backward shift operator z^{-1} and d is a white Gaussian noise sequence with unit covariance.

The input excitation is chosen as a Schroeder-phased multisinusoidal input design [5],

$$u_s(k) = \beta \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i) \tag{2}$$

where $\omega_i = 2\pi i/T_p$, $T_p/T = N_s$, $n_s \leq N_s/2$. The power is normalized as $\sum_{i=1}^{n_s} \alpha_i = 1$ where the relative power in each component $\{\alpha_i > 0, i = 1, \ldots n_s\}$ is assumed specified. In order to minimize peaking in time domain the sinusoids are phased according to Schroeder [5][29] as,

$$\phi_i = 2\pi \sum_{j=1}^{i} j\alpha_j \tag{3}$$

At steady-state, the plant response to u_s is denoted as y_s and is given by,

$$y_s(k) = \sum_{i=1}^{n_s} (b_i\beta\sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i) - a_i\beta\sqrt{2\alpha_i} \sin(\omega_i kT + \phi_i)) + v(k) \tag{4}$$

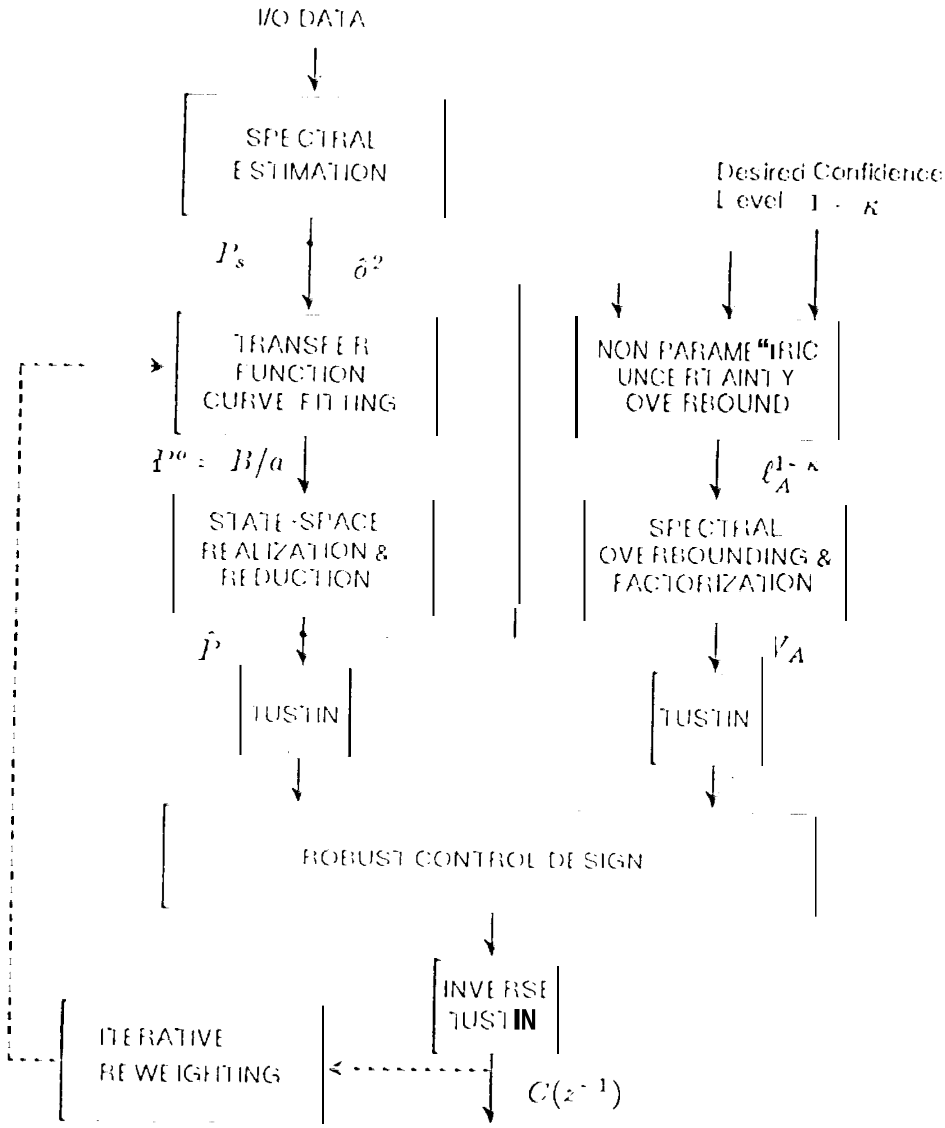

Figure 2: Identification and robust control integration

where,

$$a_i = \Im\{P(e^{-j\omega_i T})\}, \quad b_i = \Re\{P(e^{-j\omega_i T})\} \tag{5}$$

For notational convenience, the index k starts from (1 in (4) even though we are in steady-state. Since the goal is to estimate the quantities a_i and b_i it is convenient to collect these quantities in a single vector θ defined as follows,

$$o = [a^T, b^T]^T \tag{6}$$

$$a = [a_1, \ldots [1.,'_{1'}; \quad b = [b_1, \ldots, b_{n_s}]^T \tag{7}$$

Let the noise transfer function be decomposed as,

$$W_d(z^{-1}) = o\bar{W}(z^{-1}) \tag{8}$$

Methods for plant estimation will depend upon whether the noise parameters $\bar{W}$ and/or o are known or unknown. These cases are considered separately below for SISO plants. More general expressions applicable to MIMO plants can be found in [8].

2.1 Case 1a : $\bar{W}$ and o Known

If filter $\bar{W}$ is known one can "whiten" the effect of the noise in (4) by inverse filtering,

$$\bar{W}(z^{-1})\hat{y}_s(k) = y_s(k); \quad \bar{W}(z^{-1})\hat{u}_s(k) = u_s(k) \tag{9}$$

Assume that m periods of filtered input/output data $\hat{u}_s$, $\hat{y}_s$ are collected at steady-state. Denote the output data from the ℓth period as,

$$\hat{y}_s^\ell(k) = \hat{y}_s(k + (\ell - 1)N_s) \tag{10}$$

for $k = 0, \ldots, N_s - 1$ and $\ell = 1, \ldots, m$.

Construct plant estimate P_s by averaging DFT's as follows,

$$P_s(\omega_i) = \frac{\frac{1}{m}\sum_{\ell=1}^{m} \hat{Y}_s^\ell(\omega_i)}{\hat{U}_s(\omega_i)} \tag{11}$$

$$\hat{a}_i = \Im\{P_s(\omega_i)\}, \quad \hat{b}_i = \Re\{P_s(\omega_i)\} \tag{12}$$

where,

$$\hat{Y}_s^\ell(\omega_i) = \sum_{k=0}^{N_s-1} \hat{y}_s^\ell(k)e^{-j\omega_i kT}; \quad \hat{U}_s(\omega_i) = \sum_{k=0}^{N_s-1} \hat{u}_s(k)e^{-j\omega_i kT} \tag{13}$$

Assuming o is *known*, and $m > 1$ windows of data are taken in steady-state, the *exact* error probability distributions are given as,

$$\hat{o} = [ii], \ldots, \hat{a}_{n_s}, \hat{b}], \ldots, \hat{b}_{n_s}]^T \tag{14}$$

$$\frac{|\mathcal{P}(e^{-j\omega_i T}) - P_s(\omega_i)|^2}{\sigma^2 c_{ii}} \sim \chi^2(2); \quad \theta - \hat{\theta} \sim N(0, \Sigma) \tag{15}$$

$$\Sigma = \sigma^2 \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} \tag{16}$$

$$C = diag[c_{11}, \ldots, c_{n_s n_s}]; \; c_{ii} = |W(e^{-j\omega_i T})|^2/(\beta^2 \alpha_i m N_s) \tag{17}$$

where $N(\bar{x}, X)$ is a multivariate Normal distribution with mean $\bar{x}$ and covariance X, and $\chi^2(\nu)$ denotes a Chi-Squared distribution with ν degrees of freedom.

2.2 Case 2: W Known, σ Unknown

If σ is unknown, it can be estimated as follows,

$$\hat{\sigma}^2 = \frac{\sum_{\ell=1}^{m} \sum_{i=0}^{N_s-1} |\hat{Y}_s^{\ell}(\omega_i) - \bar{Y}_s(\omega_i)|^2}{N_s(mN_s - 2n_s)} \tag{18}$$

where,

$$\bar{Y}_s(\omega_i) = \begin{cases} \frac{1}{m} \sum_{\ell=1}^{m} \hat{Y}_s^{\ell}(\omega_i) & \text{for } \omega_i \text{ or } \omega_{N_s - i} \text{ in } u_s \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

It is noted that the summation in (18) is over the two-sided DFT spectrum, and that $\bar{Y}_s$ in (19) is zeroed out at all harmonic components which have zero energy in u_s. This general formula allows the designer freedom to exclude harmonics in the multisinusoidal sum (2).

If σ is estimated using (18), and m windows of data are taken in steady-state, the *exact* error probability distributions are given as,

$$(mN_s - 2n_s)\frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2(mN_s - 2n_s) \tag{20}$$

$$\frac{|\mathcal{P}(e^{-j\omega_i T}) - P_s(\omega_i)|^2}{2\hat{\sigma}^2 c_{ii}} \sim F(2, mN_s - 2n_s) \tag{21}$$

where $F(\nu_1, \nu_2)$ denotes a Fisher distribution with ν_1 and ν_2 degrees of freedom, and $t(\nu)$ denotes a Student t distribution with ν degrees of freedom.

The $(1-\alpha) \times 100\%$ confidence region for the case of σ^2 estimated by $\hat{\sigma}^2$ is a perfect circle centered at $P_s(\omega_i) = \hat{b}_i + j\hat{a}_i$ of radius r_i where from (21),

$$r_i^2 = \frac{2\hat{\sigma}^2 |W(e^{-j\omega_i T})|^2 F_{1-\alpha}(2, mN_s - 2n_s)}{\beta^2 \alpha_i m N_s} \tag{22}$$

2.3 Case 3: W Unknown, σ Unknown

If the noise transfer function $W_d = \sigma W$ is completely unknown, it can be estimated using an approximate (asymptotic) analysis as given in this section.

For this purpose, the following assumption is made,

Assumption 1 Assume that the length of the data window $T_p = TN_s$ is large compared to the time constants of $\bar{W}$ and its inverse $\bar{W}^{-1}$. ∎

Since u_s is a periodic function, it can be shown that $\hat{U}_s(\omega_i) = U_s(\omega_i)/\bar{W}(e^{-j\omega_i T})$ (i.e., inverse filtering by $\bar{W}$ is equivalent to dividing by $\bar{W}$ in frequency domain). This property follows directly from the well-known correspondence between circular convolutions and multiplication of DFT's, and in general will not hold for non-periodic signals. However, in light of Assumption 1 and (9), a similar relation is approximately true for the output, i.e., $\hat{Y}_s^\ell(\omega_i) \simeq Y_s^\ell(\omega_i)/\bar{W}(e^{-j\omega_i T})$. Substituting these expressions into (11) gives,

$$P_s(\omega_i) = \frac{\frac{1}{m}\sum_{\ell=1}^{m} \hat{Y}_s^\ell(\omega_i)}{\hat{U}_s(\omega_i)} \simeq \frac{\frac{1}{m}\sum_{\ell=1}^{m} Y_s^\ell(\omega_i)}{U_s(\omega_i)} \tag{23}$$

Note that the dependence on $\bar{W}$ divides out in (23) to give computations completely in terms unfiltered quantities. Hence, results (11)-(17) of Section 2.1 hold with (11) replaced by (23). This observation is crucial since the filter $\bar{W}$ is assumed to be unknown in the present case. In this case, the plant can be estimated by the ratio of DFT's of unfiltered quantities,

$$P_s(\omega_i) = \frac{\frac{1}{m}\sum_{\ell=1}^{m} Y_s^\ell(\omega_i)}{U_s(\omega_i)} \tag{24}$$

where,

$$Y_s^\ell(\omega_i) = \sum_{k=0}^{N_s-1} y_s^\ell(k)e^{-j\omega_i kT}; \quad U_s(\omega_i) = \sum_{k=0}^{N_s-1} u_s(k)e^{-j\omega_i kT} \tag{25}$$

Let $m = 1$ in the expressions (15)-(17), to give statistics of the plant estimate θ^ℓ obtained using the ℓth data window processed alone, i.e.,

$$\hat{\theta}^\ell \sim N(\theta, \Sigma^1) \tag{26}$$

$$\Sigma^1 = \sigma^2 \begin{pmatrix} C^1 & 0 \\ 0 & C^1 \end{pmatrix} \tag{27}$$

$$C^1 = diag[c_{11}^1, \ldots, c_{n_a n_a}^1]; \quad c_{ii}^1 = |\bar{W}(e^{-j\omega_i T})|^2/(\beta^2\alpha_i N_s) \tag{28}$$

Here, the superscript "1" denotes that $m = 1$, i.e., only a single window is used in the computation.

Since the statistics for each window are Gaussian and given by (26) (with diagonal covariance matrix), one can write the spectral estimate at each frequency grid point as,

$$\hat{\theta}_i^\ell \sim N(\theta_i, \sigma^2 c_{ii}^1 I) \tag{29}$$

where,

$$\hat{\theta}_i^\ell = \begin{pmatrix} \hat{a}_i^\ell \\ \hat{b}_i^\ell \end{pmatrix}; \quad \theta_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix} \tag{30}$$

$$P_s^\ell(\omega_i) = \frac{Y_s^\ell(\omega_i)}{U_s(\omega_i)} \tag{31}$$

$$\hat{a}_i^\ell = \Im\{P_s^\ell(\omega_i)\}, \quad \hat{b}_i^\ell = \Re\{P_s^\ell(\omega_i)\} \tag{32}$$

By Assumption 1, each data window is long compared to the time constants of W and hence the data windows are approximately statistically independent. By noting that the spectral estimate ('24) is an average of m (approximately) independent normal variates, one can invoke Normal the'c)ry to generate statistics of the mean an c1 covari ance in (29). This gives the following noise estimate,

$$|\hat{W}_d(\omega_i)|^2 = \frac{\sum_{\ell=1}^{m} |\hat{Y}_s(\omega_i) - Y_s^\ell(\omega_i)|^2}{(m-1)N_s} \tag{33}$$

where,

$$\hat{Y}_s(\omega_i) = \begin{cases} \frac{1}{m}\sum_{\ell=1}^{m} Y_s^\ell(\omega_i) & \text{for } \omega_i \text{ or } \omega_{N_s-i} \text{ in } u_s \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

Given Assumption 1, the error probability distributions are given *approximately* as,

$$\frac{\beta^2\alpha_i m N_s |P(e^{-j\omega_i T}) - P_s(\omega_i)|^2}{2|\hat{W}_d(\omega_i)|^2} \sim F(2, m-1) \tag{35}$$

$$\frac{2(m-1)|\hat{W}_d(\omega_i)|}{|W_d(e^{-j\omega_i T})|} \sim \chi^2(2(m-1)) \tag{36}$$

The $(1-\alpha) \times 100\%$ confidence region for the case of W_d estimated by $\hat{W}_d$ is a perfect circle centered at $P_s(\omega_i) = \hat{b}_i + j\hat{a}_i$ Of radius ϵ_i where from (35),

$$\epsilon_i^2 = \frac{2|\hat{W}_d(\omega_i)|^2 F_{1-\alpha}(2, m-1)}{\beta^2\alpha_i m N_s} \tag{37}$$

2.4 Transfer Function Curve Fitting

Consider the problem of finding a rational transfer function $P^o(z^{-1}) = b(z^{-1})/a(z^{-1})$ whit.}1 minimizes the weighted 2-norm of the! error between itself and specified frequency domain data P_s, i.e.,

$$\min_{P^o} \sum_{i=1}^{n_s} w^2(\omega_i) \left| P_s(\omega_i) - P^o(e^{-j\omega_i T}) \right|^2 \tag{38}$$

A simple but approximate algorithm [27] is given by the! following fixed-point iteration, (denoted here as the SK iteration),

$$a^{k+1}, b^{k+1} = arg \min_{a,b} \sum_{i=1}^{m} w^2(\omega_i) \left| \frac{1}{a^k(e^{-j\omega_i T})} \left(P_s(\omega_i) a(e^{-j\omega_i T}) - b(e^{-j\omega_i T}) \right) \right|^2 \tag{39}$$

with initial condition $a^0 = 1$, $b^0 = 0$. With a^k fixed at each iteration, the cost function in (39) is quadratic. in the coefficients of a and b. Hence, the SK iteration is implemented as a sequence of linear least squares problems.

The original work of Sanathanan and Koerner [27] is formulated in the Laplace s domain. Details of the formulation in the z-domain with some practical improvements and extensions to multivariable systems cam be found in [3],

2.5 State-Space Realization

Given P^o, one can divide. $a(z^{-1})$ into $b(z^{-1})$ to give the Markov parameter sequence $\{h_i\}$. A balanced state-space realization is determined from the Markov parameters $\{h_i\}$ using any one of a number of realization methods based on the singular-value decomposition [18][21]. With this approach, the model reduction is performed systematically in terms of the Hankel singular values, and leads to a desired reduced-order balanced state-space model $\hat{P}$ with realization (A, B, C, D).

The identification approach defined by combining the curve fitting step in Section *2.4* and realization step in Section 2.5 has certain advantages over other existing frequency domain methods, and is discussed in more detail in [4].

2.6 Nonparametric Overbounds

The error $P - \hat{P}$ at each grid point can be overbounded by using the following inequality,

$$|\Delta_A(\omega_i)| : |P(\omega_i) - \hat{P}(e^{-j\omega_i T})| < |P(\omega_i) - P_s(\omega_i)| + |P_s(\omega_i) - \hat{P}(e^{-j\omega_i T})| \tag{40}$$

The first term on the right hand side of (40) is probabilistic and can be overbounded to any desired confidence using the statistics of spectral estimation error (cf., (2.?.) or (37)). The second term on the right hand side can be calculated exactly since P_s and $\hat{P}$ are known.

In this manner, a statistical overbound $\ell_A^{1-\kappa}$ on the uncertainty can be computed, i.e.,

$$\ell_A^{1-\kappa}(\omega_i) \geq |\Delta_A(\omega_i)| \tag{41}$$

with probability $1 - \alpha$ for each $i = 1, \ldots, n_s$. Then the probability of overbounding all n_s data points *simultaneously* is given by [5],

$$1 - \kappa = \begin{cases} (1-\alpha)^{n_s} & \text{for independent errors} \\ 1 - \alpha n_s & \text{for dependent errors} \end{cases} \tag{42}$$

With this construction, $\ell_A^{1-\kappa}$ is an overbound on the additive uncertainty set at all grid points ω_i $i = 1, \ldots, n_s$ simultaneously with at least probability $1 - \kappa$.

When using (42), the errors at each grid point cam always be treated as dependent. However, in light of Assumption 1, the choice $1 - \kappa = (1 - \alpha)^{n_s}$ can be used with little error since the frequency domain processing tends to make the errors independent.

The above analysis only ensures overbounding at the grid points. Overbounding *in-between* grid points can be done using a-priori estimates of the damping in the system, as shown in [5]. However, this interpolation error will be ignored in the present analysis since such theoretical expressions are overly conservative for lightly damped systems. Rather, it is assumed that reasonable engineering judgement has been made in the choice of frequency grid so that the interpolation error can be ignored. (Such grids are required anyway for most other engineering analysis, plots, etc.).

2.7 Spectral Overbounding and Factorization

The LPSOF algorithm introduced in [28] is used for determining a minimum-phase transfer function W_A such that $|W_A|$ is a tight overbound on $\ell_A^{1-\kappa}(\omega_i)$ $i = 1, \ldots, n_s$.

Forming the. quantity $W_A(z)W_A(z^{-1})$ and evaluating on the unit circle gives an expression of the form,

$$W_A^* W_A = \frac{\beta(\omega)}{\alpha(\omega)} \tag{43}$$

where,

$$\beta(\omega) = \beta_0 + \beta_1 cos(\omega T) + . .. + \beta_m cos(m\omega T) \tag{44}$$

$$\alpha(\omega) = 14 \ \alpha_1 cos(\omega T) + ... + \alpha_m cos(m\omega T) \tag{45}$$

The requirement that $|WA|$ be an overbound on some specified function of frequency $\ell(\omega)$ is equivalent to the requirement that $|W_A|^2$ is an overbound 011 the! square of the function ℓ^2 and can be expressed as,

$$\frac{\beta(\omega)}{\alpha(\omega)} \geq \ell^2(\omega) \ for \text{ all } \omega \in [0, X/T] \tag{46}$$

The requirement that $|W_A|^2$ be a "tight" overbound can be expressed as,

$$\min_{\alpha,\beta} \delta \tag{47}$$

wh ere,

$$\delta = \max_{\omega} \{ (\frac{\beta(\omega)}{\alpha(\omega)} - \ell^2(\omega)) q^{-1}(\omega) \} \tag{48}$$

here, the criterion minimizes a worst- case error δ, which is fi equency weighted by the quantity $q^{-1}(\omega)$. The requirement that the. overbound β/α admits spectral factor WA cam be satisfied by ensuring that (cf.) Astrom [1]),

$$\beta(\omega)/\alpha(\omega) > 0 \ for \text{ all } \omega \in [0,77/7'] \tag{49}$$

$$\alpha(\omega) > 0 \ jor \ all \ \omega \in [0, \pi/T] \tag{50}$$

Note that condition (49) is implied by (46), and condition (50) can be enforced explicitly by the constraint ,$\alpha(\omega) \geq \underline{\alpha} > 0$ for some small $\underline{\alpha}$. For technical reasons, a similar constraint is enforced on β as $\beta(\omega) \geq \underline{\beta} > 0$ for some small $\underline{\beta}$. The constrained optimization problem above can be written on the frequency grid as,

$$\min_{\delta,\alpha_j,\beta_j} \delta \tag{51}$$

subject to

$$\beta(\omega_i) - \ell^2(\omega_i)\alpha(\omega_i) \geq 0 \tag{52}$$

$$\beta(\omega_i) - \ell^2(\omega_i)\alpha(\omega_i) \leq \delta q(\omega_i)\alpha(\omega_i) \tag{53}$$

$$\beta(\omega_i) > \underline{\beta}; \ \alpha(\omega_i) > \underline{\alpha} \tag{54}$$

$$for \ all \ \omega_i, i = 1,..., n_s$$

where $\alpha(\omega)$ and $\beta(\omega)$ are. defined by (44). A key observation is that for fixed δ the optimization over c y, β is simply a linear programming problem to fin d a *feasible solution* for the coefficients $\alpha_i \beta_i$. Hen ce, the joint optimization problem can be solved by a nested search procedure where an outer-loop systematically decreases δ while an inner-loop finds feasible solutions in the variables α and β for fixed δ. This algorithm is converges to the globally *optimal solution* of the nonlinear constrained minimax problem defined by (51)(52)(53)(54) [28].

Once the solution β/α is found, it can be spectrally factored by factoring $\beta = b^*b$ and $\alpha = a^*a$ separately as polynomials. We then choose $W_A = b/a$. An excellent algorithm for polynomial factorization which avoids solving for roots, is given in Kucera [2.0].

3 Robust Control Design

Once the nominal plant $\hat{P}(z)$ and additive uncertainty AA $= \Delta W_A(z)$ have been characterized using the methods above, it becomes a well-known problem in the. robust control literature to frnd a controller with desired robustness properties. For example, there is a gene.ral framework for robust control synthesis based on the. μ measure [15][14] which can be invoked at this point. Alternatively, the discussion here will concentrate on an H_∞ approach based on the weighted mixed-sensitivity H_∞ problem.

For use with software packages such as [2] [1 1] which are applicable to s-domain control design, it is necessary to use a Tustin transformation (cf., [1 1]) to convert the "z" plane quantities $\hat{P}(z)$ and $W_A(z)$ to the "w" plane for control design. An inverse Tustin transformation is then used to convert the control design back to sampled-data form for implementation. Since the H_∞ norm is invariant under the Tustin transformation, robustness and performance bounds satisfied by the controller in the w plane will carry over when implemented in the z domain.

3.1 Weighted Mixed Sensitivity problem

It can be shown that a necessary and sufficient condition for robust performance (with implied robust stability) can be written with respect to weighting WA as,

$$\mu(T) < 1, \quad \text{for all } \omega \tag{55}$$

where,

$$T = \begin{bmatrix} \gamma W_1 S(\hat{P}, C) & -\gamma W_1 S(\hat{P}, C) \\ W_A Q(\hat{P}, C) & -W_A Q(\hat{P}, C) \end{bmatrix} \tag{56}$$

$$S(\hat{P}, C) = (I + \hat{P}C)^{-1}; \quad Q(\hat{P}, C) = C(I + \hat{P}C)^{-1} \tag{57}$$

Here, μ is the structured singular value introduced by Doyle [15][1 4]. To find the controller which provides the best robust pcrforJr-rant.c from (55) (i.e., μ-synthesis), it is require.d to maximize 7 over choice of C. This is typic. ally done in an iterative. manner, by first fixing 7 and finding a feasible solution to (55). If one exists, the. value of 7 is increased and a new feasible solution is sought. The largest value of γ which admits a feasible solution C is optimal. Methods for synthesizing robust controllers based on p-synthesis are given in [2].

For the purposes of this paper, control design will be considered in terms of a related H_∞ criteria for robust perform ante. In particular, it can be shown that the following inequalities hold [1 O],

$$||J||_\infty \leq \sup_\omega \mu(T) \leq \sqrt{2}||J||_\infty \tag{58}$$

Where,

$$J = \begin{bmatrix} \gamma W_1 S(\hat{P}, C) \\ W_2 Q(\hat{P}, C) \end{bmatrix} \quad (59)$$

and $W_2 = WA$ for the present application. This implies that designing a compensator to maximize γ subject to $||J||_\infty < 1$ will give a result to within 3db of the μ-synthesis approach. Finding such a C is called the *weighted mixed-sensitivity H_∞ problem* in the literature, and can be solved using available software [11].

4 Numerical Example

A 50 state model is used for simulation purposes (obtained from the JPL ARC identification experiment [12]). The noise W_d is chosen as a first order lowpass noise filter with 16 Hz bandwidth rolloff. Two hundred windows (i.e., $m = 200$), of the steady-state response to a flat Schroeder design (2), ($T = .005$ seconds, $n_s = 256$, $\alpha_i = 1$), are averaged and quantities P_s, $|\hat{W}_d|$ and ϵ_i (to 95% confidence) are generated using formulas (24)(33)(37), respectively. The estimate P_s and its error ϵ_i are superimposed in Figure. 3 (a). The estimate $|W_d|$ is shown in Figure 3 (b) superimposed on the true $|W_d|$ for comparison, It is seen that the correspondence is excellent.

The estimate P_s is then curve fitted using the method in Section 2,4 and realized in state-spat.c form using the method in Section 2,5, The realized model $\hat{P}$ and the fitting error $|\hat{P} - P_s|$ are superimposed in Figure 3 (c). Finally, the total error $\ell_A^{1-\kappa} = |\hat{P} - P_s| + \epsilon$ is shown as the da.shed line in Figure 3 (d), and overbounded with a 3rd order WA determined by the LPSOF algorithm of Section 2.'7. The overbound WA is plotted with a solid line in Figure 3 (d) and is seen to be very tight as desired.

The H^∞ synthesis software (**hinf.m** [11]), is used to find a discrete stabilizing, feedback controller $C(z)$ to minimize the discrete H^∞ norm of the cost function (56) (the. design is computed directly in discrete-time and no Tustin transformation is required),

The weighting function WI penalizes the system for disturbance rejection. The present design goal is to suppress the first 3 low-damped modes with direct H^∞ control. These three modes are at 8, 10.5, and 11 Hz. A modal truncation M-file (**modreal.m** in [11]) gives a 6-state truncated model of' the original 50-state identified model. This 6-state model is used as the weighting function WI (see Figure 4(c)). The weighting function W_2 is taken as the uncertainty overbound realization WA.

If such an H^∞ Controller exists, it will make the cost function close to all-pass i.e., it will make the sensitivity function inverse to WI, and CS inverse to W_2, thereby rejecting disturbances around those pre-specified modes in the presence of plant additive uncertainty and achieving the *robust stability* and *robust performance* design goal.

Figure 4 shows the H^∞ design. The responses are all well below the inverse weighting functions as desired, and the performance index is $\gamma = .23$. Figure 5 shows the open and closed-loop transfer function associated with the *true* plant. The true closed-loop system is stable and de.sired vibration suppression of the first 3 modes is achieved,

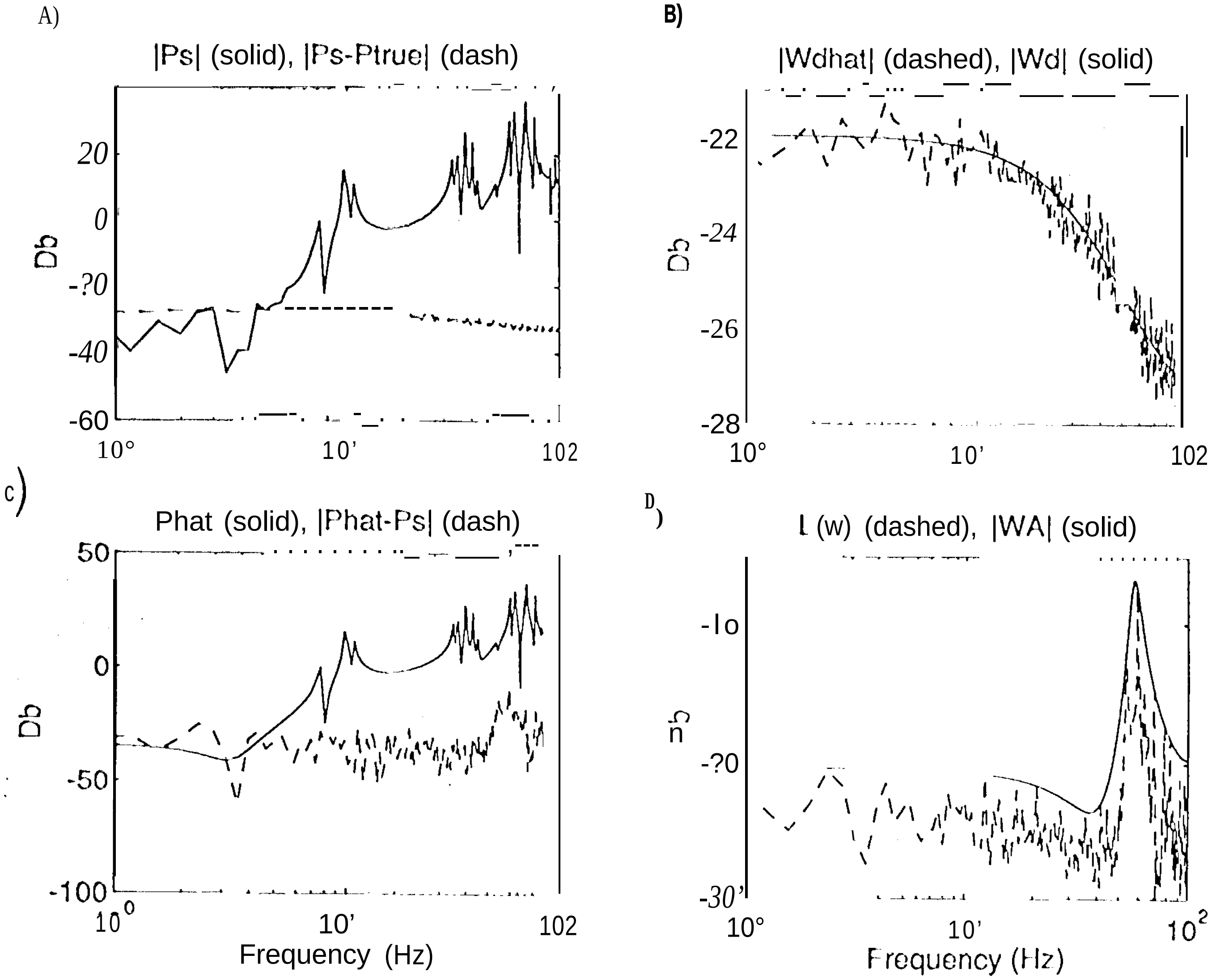

Figure 3: Identification results

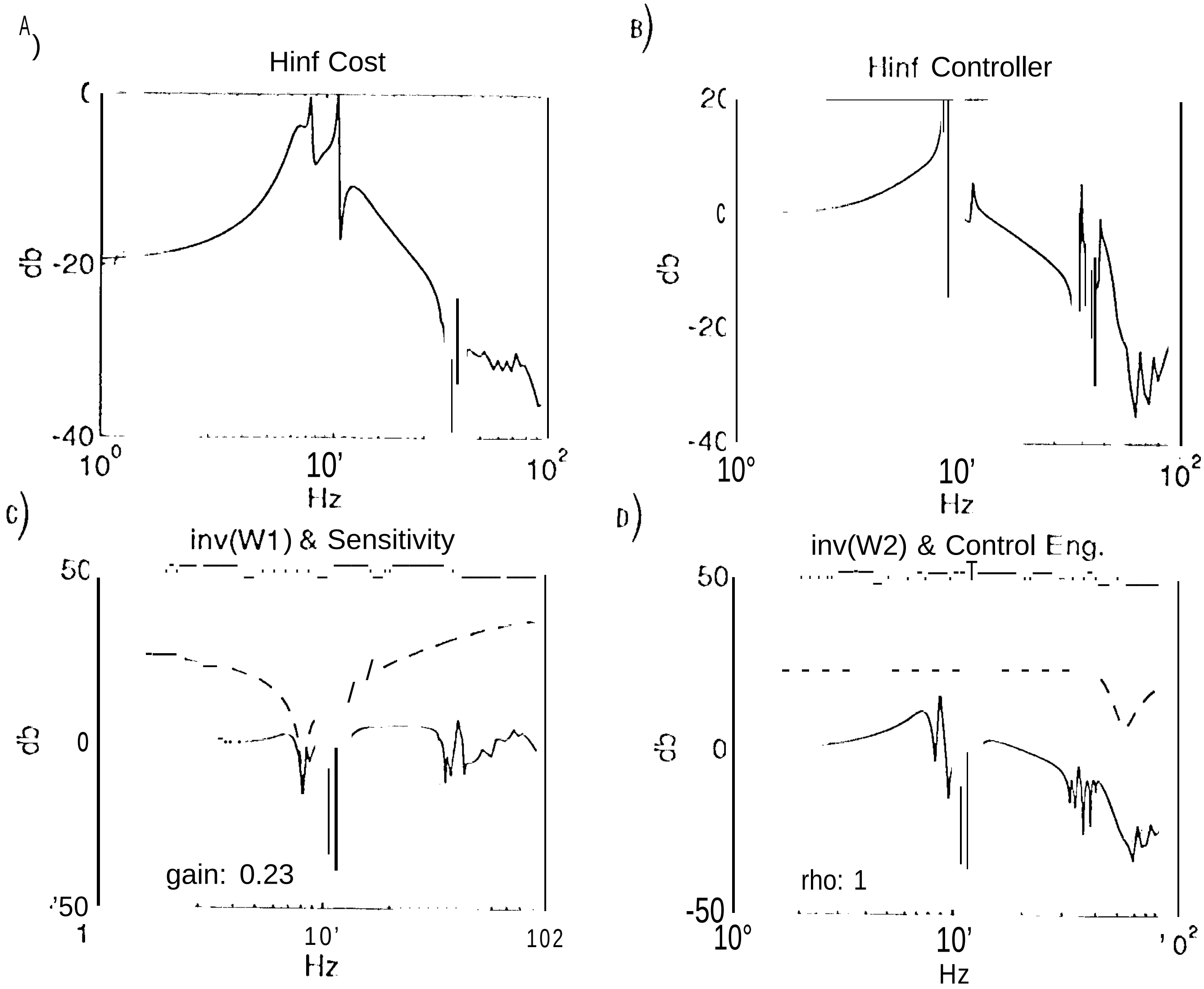

Figure 4: Discrete $\mathbf{H}^{\infty}$ Design,

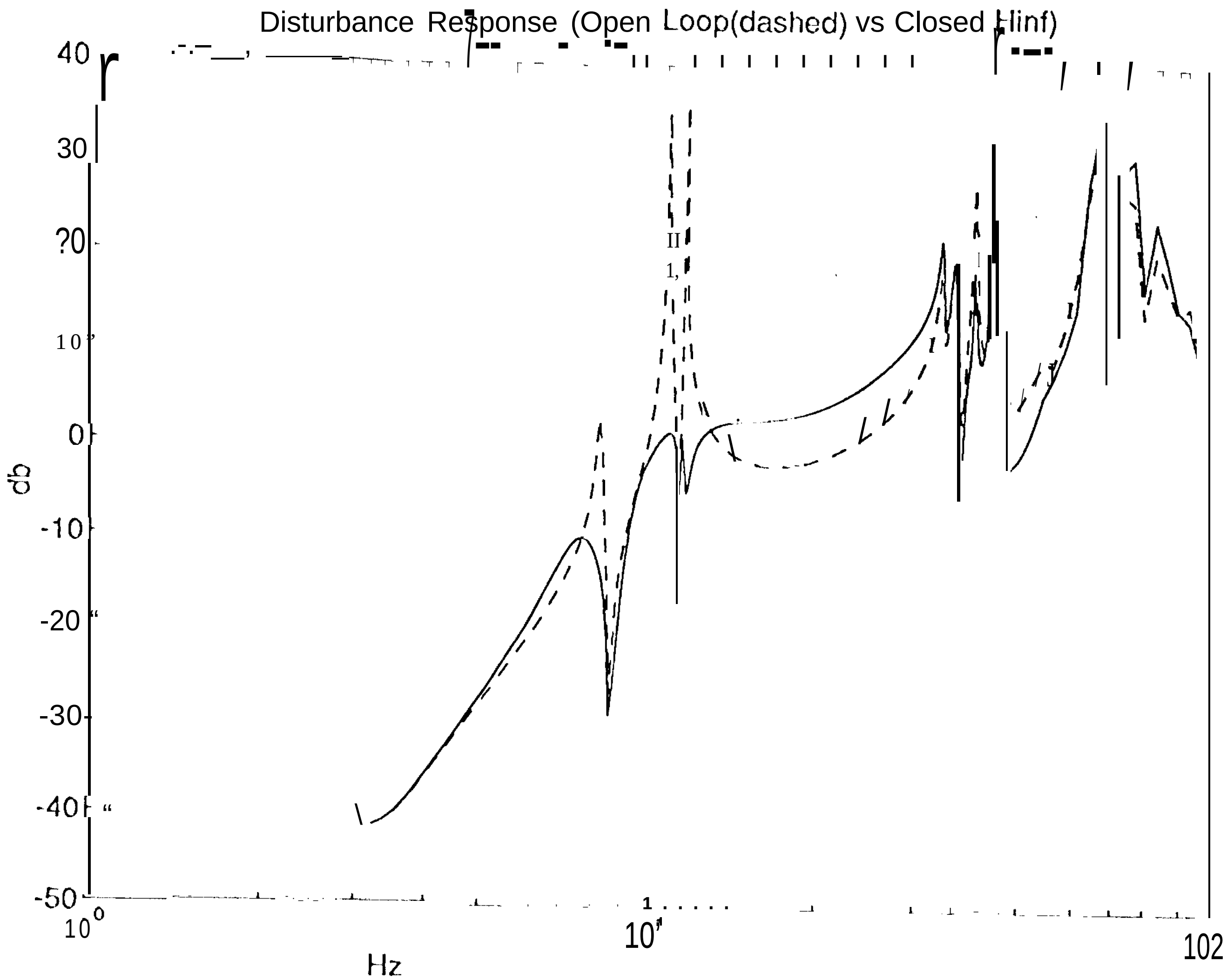

Figure 5: Discrete disturbance response (Dashed: Open Loop; Solid: Closed Loop)

5 Conclusions

Simply stated, the results of this paper allow one to design a robust controller from raw experimental data, Specifically, a frequency domain method identifies both a tin-m-invariant plant (in state-space form) and its uncertainty bounds. The main usefulness of this approach is that the bounds are. identified as a parametric weighting on the additive uncertainty which can be used directly for determining a robust control design. The uncertainty bounds can be determined to any specified statistical confidence, leading to a robust control design which will work as designed on the true system to the same statistical confidence. This approach is a special S1S0 case of the more general MIMO method put forth in [6][9].

A numerical example was given to demonstrate the design of a robust controller for a lightly damped large flexible structure to a prescribed 95% *confidence led.* As expected from the theory, the. controller worked as designed when connected to the true plant model.

A main simplifying assumption is that the unknown plant is linear time-invariant (1,1'1). While the method is not strictly valid when this condition is violated, there are some cases when it can be suitably modified. For example, if the plant has slowly time-varying dynamics which can be characterized by separate means, the same scheme can be used by simply augmenting Figure 1 with additional uncertainty blocks.

Acknowledgements

This research was performed at the Jet }'repulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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